Electron to Muon Conversion in Electron-Nucleus Scattering as a Probe of Supersymmetry

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We suggest that $e \to \mu$ conversion in low-energy electron-nucleus scattering is a new and potentially observable indirect signal of supersymmetry and should be searched for in experiment. We estimate the rate for this process in a straightforward calculation taking into account the existing constraint from non-observation of the $\mu \to e\gamma$ decay and find the cross-section to be $\sigma < 10^{-8}$ fb.

Introduction

The recent advances in neutrino physics have established that not only do neutrinos have mass, but also they mix strongly, thereby violating the separate lepton numbers L_e, L_μ, L_τ which, if preserved, would forbid such Lepton Flavour Violating (LFV) processes as $\mu \to e\gamma$ [1]. On the other hand, in the simplest extensions of the Standard Model, augmented to accomodate neutrino mass and mixing, the rate of such LFV processes is exceedingly small, being suppressed by the ratio of neutrino mass to W boson mass, rendering $\mu \to e\gamma$ effectively unobservable.

In the minimal supersymmetric standard model (MSSM), extended to accomodate neutrino mass and mixing, the situation is dramatically improved since any LFV in the lepton sector tends to become transferred to the slepton sector, via radiative corrections, leading to unsuppressed LFV rates which depend on ratios of superpartner masses. This has led to a considerable amount of activity associated with LFV in the MSSM and related models [1].

Much of the interest in LFV has focussed on limits arising from the classical decay experiments such as $\mu \to e\gamma$. Other experiments, usually referred to as $\mu - e$ conversion, look for LFV arising from low energy muons being captured in atoms, where the captured muon may subsequently convert to an electron whose characteristic energy is slightly below the muon mass [2].

Recently an alternative approach to searching for LFV has been proposed based on fixed-target electron scattering $e + N \to \mu + N$ at energies just above muon threshold [3]. The advantage of such a scattering experiment lies in the simplicity of both the experiment itself and the theoretical description which, unlike $\mu - e$ conversion, does not depend on addressing the problems of the QED muon-nucleus bound state. The viability of such

a proposal depends on the estimate of the $e \to \mu$ conversion cross-section in fixed target electron scattering. In [3] this has been estimated in two scenarios involving the simplest extensions of the Standard Model consistent with neutrino mass, namely the case of three light Dirac neutrinos and the see-saw mechanism [4]. It was shown that even in the most optimistic scenario with very light right-handed neutrino masses, the cross-section is at most of order 10^{-30} pb, many orders of magnitude below the capabilities of experimental detection.

The purpose of this note is to present a first calculation of the matrix element for $e \to \mu$ conversion within the framework of the MSSM. Technically the calculation resembles the classical matrix element for $\mu \to e\gamma$, but differs in that the photon in $e \to \mu$ conversion is now off-shell, leading to a rather more complicated form of the matrix element. One would expect the resulting cross-section for $e + N \to \mu + N$ in the MSSM to be many orders of magnitude larger than in non-supersymmetic models, for the reasons discussed, and this is indeed the case. We find that the process of $e \to \mu$ conversion in low-energy electron-nucleus scattering provides a potentially observable indirect signal of supersymmetry, and we therefore advocate that such experiments be seriously considered.

The Amplitude for the $e \to \mu$ Conversion in Supersymmetry (SUSY)

The SUSY matrix element for the electron to muon conversion at low energy is dominated by the photon exchange with the nucleus just like the similar estimate in the Standard Model [3]. The scalar amplitude due to the Higgs exchange is suppressed not only due to the heavy Higgs masses but also due to the small Yukawa couplings. The matrix element respecting Lorentz symmetry then assumes the form

$$\langle \mu | j^\lambda(0) | e \rangle = \overline{u}(p') \ [\ ik_\nu \sigma^{\lambda\nu}(A + B\gamma_5) + \gamma^\lambda(C + D\gamma_5)$$
$$+ \ k^\lambda(E + F\gamma_5) \ ] \ u(p) \qquad (1)$$

where the electron, muon and photon momenta are p, p' and k, respectively, with $p = p' + k$. Two of the six

formfactors above can be eliminated due to electric current conservation, $k_\lambda j^\lambda = 0$. For the on-shell electron and muon, and neglecting the electron mass, the matrix element (1) can then be reexpressed as

$$
\begin{aligned}
\langle \mu | j^\lambda(0) | e \rangle = \quad & F_1 \, \overline{u}(p') \left[\gamma^\lambda + k^\lambda \frac{m_\mu}{k^2} \right] u(p) \\
+ \; & F_2 \, \overline{u}(p') \left[\gamma^\lambda + k^\lambda \frac{m_\mu}{k^2} \right] \gamma_5 \, u(p) \\
+ \; & F_3 \, \overline{u}(p') \left[\gamma^\lambda - \frac{(p+p')^\lambda}{m_\mu} \right] u(p) \\
+ \; & F_4 \, \overline{u}(p') \left[\gamma^\lambda - \frac{(p+p')^\lambda}{m_\mu} \right] \gamma_5 \, u(p),
\end{aligned}
\tag{2}
$$

where the formfactors $F_1 = C = -(k^2/m_\mu)E$, $F_2 = D = (k^2/m_\mu)F$, $F_3 = m_\mu A$ and $F_4 = -m_\mu B$ originate from loop diagrams that violate lepton flavour and depend on the particle masses involved in the loops. If low-energy supersymmetry is the answer to the hierarchy problem the formfactors emerge at the scale $\mathcal{O}(100\,\mathrm{GeV})$. Above this scale the effective lagrangian is the MSSM lagrangian with small lepton-flavour violating terms whose origin is explained in a more complete theory at a scale $\mu_{LFV} \gg 100\,\mathrm{GeV}$. The relevant loops contain chargino-sneutrino, neutralino-selectron, and lepton-Higgs exchanges.

In our calculation presented below we neglect the Yukawa couplings of the electron and muon in order to obtain a simple and transparent result for the formfactors F_i. Furthermore, to focus on the single dominant contribution we neglect the hypercharge gauge coupling and off-diagonal chargino and neutralino masses. We thus identify the wino-slepton loop as the dominant contribution to the electron-to-muon conversion. This approximation will provide an excellent estimate in the case when the higgsino mass (the μ term) is much greater than the gaugino mass M_2, say $M_2 = \mathcal{O}(100\,\mathrm{GeV})$ and $\mu = \mathcal{O}(1\,\mathrm{TeV})$, as often happens in the low $\tan\beta$ regime in the constrained MSSM. It also reduces the number of the independent formfactors since it is the left electrons that dominate this contribution and we can write $\langle \mu | j^\lambda(0) | e \rangle = F_+ \mathcal{M}_+ + F_- \mathcal{M}_-$ where $F_+ = F_1 = -F_2$, $F_- = F_3 = -F_4$, $\mathcal{M}_+ = \overline{u}(p') \left[\gamma^\lambda + k^\lambda \frac{m_\mu}{k^2} \right] u_L(p)$ and $\mathcal{M}_- = \overline{u}(p') \left[\gamma^\lambda - \frac{(p+p')^\lambda}{m_\mu} \right] u_L(p)$. Next, we present the results for the formfactors F_+ and F_-.

The MSSM interaction of the wino-like charginos and neutralinos with charged leptons is given by

$$
\mathcal{L} = \overline{e_{iL}} \left[-g_2 \, (\Gamma^\dagger_{\nu L})_{i\alpha} \, \tilde{w}^- \, \tilde{\nu}_\alpha + \frac{g_2}{\sqrt{2}} \, (\Gamma^\dagger_{e L})_{i\beta} \, \tilde{w}^0 \, \tilde{e}_\beta \right] + h.c.,
\tag{3}
$$

where $i = 1, 2$ corresponds to the electron and muon, respectively, and the Γ matrices describe the additional slepton mixing after the sleptons have been rotated the same way as the charged leptons. The chargino loop, figure 1a, with the photon attached to the fermionic line,

results in

$$
\begin{aligned}
F_+^{(+)} &= \frac{g^2 e}{16\pi^2} \, (\Gamma^\dagger_{\nu L})_{2\alpha} \, (\Gamma_{\nu L})_{\alpha 1} \, \frac{k^2}{m_\alpha^2} \left[\frac{1}{3} f_1 + \frac{1}{3} f_3 \right], \\
F_-^{(+)} &= \frac{g^2 e}{16\pi^2} \, (\Gamma^\dagger_{\nu L})_{2\alpha} \, (\Gamma_{\nu L})_{\alpha 1} \, \frac{m_\mu^2}{m_\alpha^2} \, [f_1],
\end{aligned}
\tag{4}
$$

while the neutralino loop, figure 1b, with the photon attached to the scalar, gives

$$
\begin{aligned}
F_+^{(0)} &= \frac{g^2 e}{16\pi^2} \, (\Gamma^\dagger_{e L})_{2\alpha} \, (\Gamma_{e L})_{\alpha 1} \, \frac{k^2}{m_\alpha^2} \left[-\frac{5}{6} f_1 + \frac{1}{6} f_3 \right], \\
F_-^{(0)} &= \frac{g^2 e}{16\pi^2} \, (\Gamma^\dagger_{e,L})_{2\alpha} \, (\Gamma_{e L})_{\alpha 1} \, \frac{m_\mu^2}{m_\alpha^2} \left[\frac{1}{6} f_1 - \frac{1}{12} f_3 \right].
\end{aligned}
\tag{5}
$$

The f_i functions are

$$
\begin{aligned}
f_1 &= \frac{1}{12(x-1)^4} \left(x^3 - 6x^2 + 3x + 2 + 6x \log x \right), \\
f_3 &= \frac{1}{2(x-1)^3} \left(x^2 - 4x + 3 + 2 \log x \right)
\end{aligned}
\tag{6}
$$

where $x = M_2^2/m_\alpha^2$, the ratio of the wino mass squared over the slepton mass squared.

It is instructive to consider the limit $m_\alpha^2 \ll M_2^2$. In this case the sum over the slepton eigenstates simplifies giving the estimate for the total SUSY contribution

$$
\begin{aligned}
F_+ &= -\frac{g^2 e}{16\pi^2} \, \frac{k^2}{M_2^2} \, \frac{1}{6} \, (m_L^2)_{21}, \\
F_- &= -\frac{g^2 e}{16\pi^2} \, \frac{m_\mu^2}{M_2^2} \, \frac{7}{72} \, (m_L^2)_{21},
\end{aligned}
\tag{7}
$$

where $F_\pm = F_\pm^{(+)} + F_\pm^{(0)}$ and $(m_L^2)_{21}$ is the off-diagonal entry in the mass matrix for the slepton doublet in the basis where sleptons have been rotated by the same rotation that diagonalises the charged leptons: $(m_L^2)_{21} = (\Gamma^\dagger_{\ell L})_{2\alpha} \, m_\alpha^2 \, (\Gamma_{\ell L})_{\alpha 1}$.

The Cross-section and Its Correlation with the $\mu \to e\gamma$ Constraint

Following [3] we write:

$$
\frac{d\sigma(e + N \to \mu + N)}{d\Omega} = \frac{(Ze)^2}{(4\pi)^2} \, \frac{1}{k^4} \, |\langle \mu | j^\lambda(0) | e \rangle|^2, \tag{8}
$$

for a pointlike nuclear charge distribution. The integration leads to the estimate

$$
\sigma(e + N \to \mu + N) \approx 10^{-1} \left(\frac{M_W}{M_2} \right)^4 \left(\frac{(m_L^2)_{12}}{M_2^2} \right)^2 \, fb \tag{9}
$$

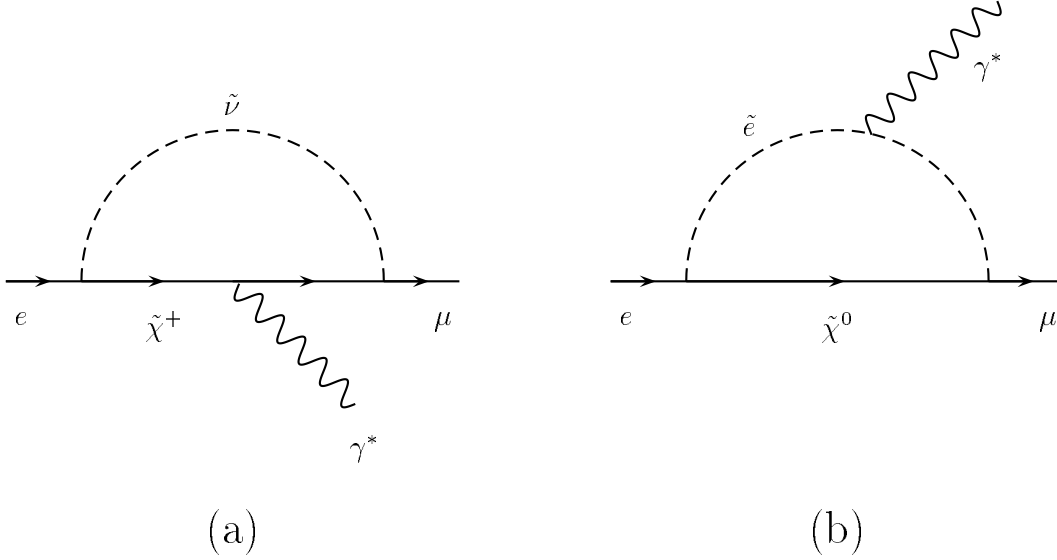in the approximation $m_\alpha^2 \ll M_2^2$, for $Z \approx 70$ and electron energy $E = 200\,\mathrm{MeV}$.

FIG. 1: Feynman diagrams for the dominant MSSM contribution to the process $e \to \mu\gamma^*$.

An upper bound on the cross-section above can be obtained from the non-observation of $\mu \to e\gamma$ decay. The MSSM branching ratio for this decay can be expressed as

$$BR(\mu \to e\gamma) = \frac{48\pi^3\alpha}{G_F^2} \, |A_R|^2. \qquad (10)$$

Neglecting the small yukawa couplings and keeping only the dominant chargino-sneutrino contribution

$$A_R = (-i) \frac{g_2^2}{16\pi^2} \frac{1}{m_\alpha^2} (\Gamma_{\nu L}^\dagger)_{1\alpha} (\Gamma_{\nu L})_{\alpha 2} \, f_1(x_\alpha), \quad (11)$$

with the same notation as in the previous section. For $m_\alpha^2 \ll M_2^2$ the branching ratio can be approximated as

$$BR(\mu \to e\gamma) \approx 10^{-4} \left(\frac{M_W}{M_2}\right)^4 \left(\frac{(m_L^2)_{12}}{M_2^2}\right)^2 \qquad (12)$$

where we have written $\frac{\alpha}{6\pi} \approx 10^{-4}$. By comparing to (9) the current experimental upper limit $BR(\mu \to e\gamma) \leq 10^{-11}$ [1] then leads to the upper bound [1]

$$\sigma(e + N \to \mu + N) \leq 10^{-8} fb. \qquad (13)$$

[1] In making this comparison we assume that $(m_L^2)_{12} = (m_L^2)_{21}$ which is valid in the case that phases are neglected.

Conclusions

We have presented a first calculation of the matrix element for $e \to \mu$ conversion in low-energy electron-nucleus scattering within the framework of supersymmetry. We have estimated the rate for this process in the MSSM, in the approximation that the wino mass parameter M_2 is much smaller than the Higgsino mass parameter μ, so that Fig.1 approximates to two wino-slepton loops, one with a charged wino and one with a neutral wino. Taking into account the existing constraint from non-observation of the $\mu \to e\gamma$ decay we find the $e \to \mu$ conversion cross-section to be approximately bounded by $\sigma < 10^{-8}$ fb. The cross-section may be orders of magnitude larger than in the simplest extensions of the Standard Model, and provides a new and potentially observable indirect signal of supersymmetry. We strongly urge our experimental colleagues to consider performing such an experiment.

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